1. List the members of these sets.

a) {x | x is a real number such that x2 = 1}  
b) {x | x is a positive integer less than 12}  
c) {x | x is the square of an integer and x < 100}  
d) {x | x is an integer such that x2 = 2}

2. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of airline flights from New York to New Delhi,  
the set of nonstop airline flights from New York to  
New Delhi  
b) the set of people who speak English, the set of people  
who speak Chinese  
c) the set of flying squirrels, the set of living creatures  
that can fly.

1. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first,or neither is a subset of the other.

a) the set of people who speak English, the set of people  
who speak English with an Australian accent.

b) the set of fruits, the set of citrus fruits.

c) the set of students studying discrete mathematics, the  
set of students studying data structures.

4. Determine whether each of these pairs of sets are equal.

a) {1, 3, 3, 3, 5, 5, 5, 5, 5}, {5, 3, 1}  
b) {{1}}, {1, {1}}

c) ∅, {∅}

5. Suppose that A = {2, 4, 6}, B = {2, 6}, C = {4, 6}, and D = {4, 6, 8}. Determine which of these sets are subsets of which other of these sets.

6. For each of the following sets, determine whether 2 is an element of that set.  
a) {x ∈ R | x is an integer greater than 1}  
b) {x ∈ R | x is the square of an integer}  
c) {2,{2}}

d) {{2},{{2}}}  
e) {{2},{2,{2}}}

f ) {{{2}}}

7. Determine whether each of these statements is true or  
false.  
a) 0 ∈ ∅ b) ∅ ∈ {0}  
c) {0} ⊂ ∅ d) ∅ ⊂ {0}  
e) {0} ∈ {0} f ) {0} ⊂ {0}  
g) {∅} ⊆ {∅}

8. Determine whether these statements are true or false.  
a) ∅ ∈ {∅} b) ∅ ∈ {∅, {∅}}  
c) {∅} ∈ {∅} d) {∅} ∈ {{∅}}  
e) {∅} ⊂ {∅, {∅}} f ) {{∅}} ⊂ {∅, {∅}}  
g) {{∅}} ⊂ {{∅}, {∅}}

9. Determine whether each of these statements is true or false.  
a) x ∈ {x} b) {x} ⊆ {x} c) {x} ∈ {x}  
d) {x} ∈ {{x}} e) ∅ ⊆ {x} f ) ∅ ∈ {x}

10. Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year

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11. Use a Venn diagram to illustrate the relationships A ⊂ B and B ⊂ C.

12. Suppose that A, B, and C are sets such that A ⊆ B and B ⊆ C. Show that A ⊆ C

13. What is the cardinality of each of these sets?  
a) {a} b) {{a}}  
c) {a, {a}} d) {a, {a}, {a, {a}}}

14. What is the cardinality of each of these sets?  
a) ∅ b) {∅}  
c) {∅, {∅}} d) {∅, {∅}, {∅, {∅}}}

15. Find the power set of each of these sets, where a and b  
are distinct elements.  
a) {a} b) {a, b} c) {∅, {∅}}  
  
16. How many elements does each of these sets have where  
a and b are distinct elements?  
a) P({a, b, {a, b}})  
b) P({∅, a, {a}, {{a}}})  
c) P(P(∅))

17. Prove that P(A) ⊆ P(B) if and only if A ⊆ B.  
  
18. Let A = {a, b, c, d} and B = {y, z}. Find

a) A × B. b) B × A.

19. What is the Cartesian product A × B, where A is the set of courses offered by the mathematics department at university and B is the set of mathematics professors at  
this university? Give an example of how this Cartesian product can be used.

20. What is the Cartesian product A × B × C, where A is the set of all airlines and B and C are both the set of all cities in the United States? Give an example of how this  
Cartesian product can be used.

21. Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. Find  
a) A × B × C. b) C × B × A.  
c) C × A × B. d) B × B × B.

22. Find A2 if  
a) A = {0, 1, 3}. b) A = {1, 2, a, b}.

23. Find A3 if  
a) A = {a}. b) A = {0, a}.

24. Translate each of these quantifications into English and  
determine its truth value.

a) ∀x∈ R (x2 = −1) b) ∃x∈ Z (x2 = 2)  
c) ∀x∈ Z (x2 > 0) d) ∃x∈ R (x2 = x)  
  
25. Find the truth set of each of these predicates where the  
domain is the set of integers.

a) P (x): x2 < 3 b) Q(x): x2 > x  
c) R(x): 2x + 1 = 0

26. Find the truth set of each of these predicates where the  
domain is the set of integers.

a) P (x): x3 ≥ 1 b) Q(x): x2 = 2  
c) R(x): x < x2

27. Let A = {1, 2, 3, 4, 5} and B = {0, 3, 6}. Find  
a) A ∪ B. b) A ∩ B.  
c) A − B. d) B − A.

28. Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}.  
Find  
a) A ∪ B. b) A ∩ B.  
c) A − B. d) B − A.

29. Show how bitwise operations on bit strings can be used to find these combinations of A = {a, b, c, d, e},B = {b, c, d, g, p, t, v}, C = {c, e, i, o, u, x, y, z}, and D = {d, e, h, i, n, o, t, u, x, y}.  
a) A ∪ B b) A ∩ B  
c) (A ∪ D) ∩ (B ∪ C) d) A ∪ B ∪ C ∪ D

30.Draw the Venn diagrams for each of these combinations of the sets A, B, and C.  
a) A ∩ (B − C) b) (A ∩ B) ∪ (A ∩ C)  
c) (A ∩ B) ∪ (A ∩ C)